XVI.—*On the proper relative Sectional Areas for Copper and Iron Lightning Rods.*—*By* R. S. Brough.

So far as mere conductivity is concerned, a comparatively thin wire of either copper or iron would suffice for the loftiest building; but such a thin conductor would be dangerous, because it would be fused by a heavy discharge of lightning.

Now the problem is to determine what relative sectional areas should be given to copper and iron rods, in order that neither should be more liable to fusion than the other.

The usual answer given, is, that an iron rod should have 4 times the sectional area of a copper rod.* This result is, I suppose, arrived at in the following way. The conductivity of copper is about 6 times as great as that of iron, but the melting point of iron is about 50 per cent. higher than that of copper, therefore $\frac{6{\cdot}0}{1{\cdot}5} = 4$, is the ratio for the sectional area of iron to copper.

This simple treatment of the problem, however, is incomplete, because it neglects to take three most important factors into consideration, namely, (1) the influence of the rise of temperature in increasing the electrical resistance of the metal, (2) the difference between the "thermal capacity" or "specific heat" of copper and iron, and (3) the fact that the iron rod being made several times more massive than the copper rod, it will require a proportionately greater quantity of heat to increase its temperature. These omissions introduce an enormous error in the result.

The effect of the passage of a discharge of lightning through the rod will be to raise its temperature.

The temperature (T) to which a given length of the rod will be raised will depend on

(1) The quantity of heat developed by the discharge.

(2) The mass of the rod.

(3) The "Specific heat" σ of the metal composing the rod.

This may be expressed mathematically as follows:

$$T = \text{Const.}\ \frac{H}{\sigma\, m}$$

where m is the mass of the unit length of the rod, which we shall assume to be uniform in sectional area throughout its length, and H is the quantity of heat developed by the discharge.

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* War Office Memorandum by Sir Fred. Chapman, R. E.

We may take $\sigma = 0{\cdot}1013$ for copper, and $= 0{\cdot}1218$ for iron. These figures were only verified by Dulong and Petit up to 300° C. It is probable, however, that their ratio, with which we are only here concerned, would not greatly alter at higher temperatures. At any rate, comparing the specific heat between 0° and 100° C, with that between 0° and 300° C, we infer that any alteration would be in favour of iron, *i. e.*, that the specific heat of iron would increase in a quicker ratio than that of copper.

Adopting the centimetre as the unit of length, the mass of one centimetre of the rod $= \rho$ a, where a is the sectional area of the rod in square centimetres, and $\rho = 8{\cdot}9$ for copper and $= 7{\cdot}8$ for iron.

Further, assuming the quantity and duration of the discharge to be constants, $\mathrm{H} = \mathrm{Const.} \times \mathrm{R}$, where R is the resistance of the unit length of the conductor.

But $\mathrm{R} = \dfrac{\lambda}{\mathrm{a}}$, where λ is the specific resistance of the metal per cubic centimetre at its temperature of fusion.

We may take the melting point of copper as 1400° C, and that of wrought iron as 2000° C* ; and, in order to find λ assume that Dr. William Siemens's formula, which he verified to 1000° C, holds good,† viz.—

$$\left.\begin{array}{l} \lambda\,t = \lambda_0\,(0{\cdot}026577\ t^{\frac{1}{2}} + 0{\cdot}0031443\ t - 0{\cdot}29751) \\ \qquad\qquad\qquad\qquad\qquad\qquad\text{for copper} \\ \lambda\,t = \lambda_0\,(0{\cdot}072545\ t^{\frac{1}{2}} + 0{\cdot}0138133\ t - 1{\cdot}23971) \\ \qquad\qquad\qquad\qquad\qquad\qquad\text{for iron} \end{array}\right\}$$

The temperature t in these formulæ is to be measured from the absolute zero, so that we have $t = 1673$ for copper, and $t = 2273$ for iron.

The value of λ_0 per cubic centimetre of copper is 1·652 Microhms, and per cubic centimetre of iron is 9·827 Microhms.‡

Thus the value of λ t per cubic centimetre of copper becomes 10 Microhms at 1673° C, and per cubic centimetre of iron becomes 107 Microhms at 2273° C.

$$\left.\begin{array}{l} \text{Hence } \mathrm{H} = \mathrm{Const.}\ \dfrac{10}{\mathrm{a}}\ \text{for copper} \\ \text{and}\quad \mathrm{H} = \mathrm{Const.}\ \dfrac{107}{\mathrm{A}}\ \text{for iron} \end{array}\right\}$$

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* Rankine's Tables.

† Bakerian Lecture, 1871.

‡ Jenkin's Cantor Lectures, from Mathiessen's experiments.

$$\text{Therefore } T = \text{Const.}\ \frac{10}{0{\cdot}1013 \times 8{\cdot}9 \times a^2} \text{ for copper}$$

$$\text{and } T = \text{Const.}\ \frac{107}{0{\cdot}1218 \times 7{\cdot}8 \times A^2} \text{ for iron}$$

$$\text{Thus } T = \text{Const.}\ \frac{11{\cdot}09}{a^2} \text{ for copper}$$

$$\text{and } T = \text{Const.}\ \frac{112{\cdot}63}{A^2} \text{ for iron}$$

Now putting $T =$ the temperature of fusion in each case

$$1400 = \text{Const.}\ \frac{11{\cdot}09}{a^2} \text{ for copper}$$

$$2000 = \text{Const.}\ \frac{112{\cdot}63}{A^2} \text{ for iron}$$

$$\begin{aligned} \text{Therefore } \left(\frac{A}{a}\right)^2 &= \frac{1400}{2000} \cdot \frac{112{\cdot}63}{11{\cdot}09} \\ &= 0{\cdot}7 \times 10{\cdot}16 \\ &\doteq 7{\cdot}112 \end{aligned}$$

$$\begin{aligned} \text{Whence } A &= 2{\cdot}7a \text{ about} \\ &= \frac{8}{3} a \text{ about} \end{aligned}$$

or the sectional area of an iron rod should be to the sectional area of a copper rod in the ratio of 8 to 3.